

Improving constraints on the neutrino mass using sufficient statistics

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ABSTRACT

We use the “Dark Energy and Massive Neutrino Universe” (DEMNUi) simulations to compare the constraining power of “sufficient statistics” with the standard matter power spectrum on the sum of neutrino masses, $M_\nu \equiv \sum m_\nu$. In general, the power spectrum, even supplemented with higher moments of the distribution, captures only a fraction of the available cosmological information due to correlations between the Fourier modes. In contrast, the non-linear transform of sufficient statistics, approximated by a logarithmic mapping $A = \ln(1 + \delta)$, was designed to capture all the available cosmological information contained in the matter clustering; in this sense it is an optimal observable. Our analysis takes advantage of the recent analytical model developed by Carron et al. (2014) to estimate both the matter power spectrum and the A -power spectrum covariance matrices. Using a Fisher information approach, we find that using sufficient statistics increases up to 8 times the available information on the total neutrino mass at $z = 0$, thus tightening the constraints by almost a factor of 3 compared to the matter power spectrum.

Key words: cosmology: large-scale-structure of the Universe, methods : numerical, methods, cosmology : cosmological parameters, methods: simulations

1 INTRODUCTION

The interest for neutrino science has been stimulated over the last decade by solar, atmospheric and accelerator experiments that showed, through the observations of the so-called “neutrino flavour oscillations” that neutrinos have a finite mass (Ahmed et al. 2004; Eguchi et al. 2003; Araki et al. 2005; McKeown & Vogel 2004). These experiments are, however, only sensitive to mass square differences between neutrino mass eigenstates, leaving the knowledge of the total neutrino mass as one of the great unsolved problems of modern physics. Cosmological probes, such as the cosmic microwave background (CMB) or large scale structures (LSS) in the Universe have the highest experimental sensitivity to date to the absolute neutrino mass, as massive neutrinos influence structure formation (Dolgov 2002; Lesgourgues & Pastor 2006, 2012).

The epoch at which massive neutrinos became non-relativistic changes the time of matter-radiation equality, increasing slightly the size of the sound horizon at recombination and thus changing the position of the CMB anisotropy

peaks. In combination with Baryon Acoustic Oscillations (BAO), measurements from the Planck satellite have provided the tightest constraints on the total neutrino mass $M_\nu < 0.17$ eV at 95% CL (Planck Collaboration et al. 2015). However this result is sensitive to the value of the Hubble parameter and including other external measurements leads to the weaker, more conservative limit of $M_\nu < 0.23$ eV. Neutrinos also impact the growth of structure by suppressing the small-scale matter density fluctuations. Thus measurements of the total matter power spectrum can improve constraints on the neutrino mass in a complementary way. The Sloan Digital Sky Survey (SDSS) DR8 LRG angular power spectrum with the WMAP7 data and a HST prior on the Hubble parameter gives $M_\nu < 0.26$ eV (95% CL) (de Putter et al. 2012). More recently Beutler et al. (2014), using the Baryon Oscillation Spectroscopic Survey (BOSS) CMASS Data Release 11 combined with measurements from the Planck satellite (without the A_L -lensing signal), weak lensing data and BAO constraints, found $M_\nu = 0.36 \pm 0.10$ eV (68% CL).

These bounds mainly use structure formation data up to the translinear regime ($k_{max} \sim 0.15 h\text{Mpc}^{-1}$), but even on such large scales some non-linear contamination

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is present and must be taken into account. Moreover, future galaxy redshift surveys such as the Dark Energy Survey¹ (DES), the Large Synoptic Survey Telescope² (LSST), Euclid³ or the Wide Field Infra-Red Survey Telescope⁴ (WFIRST), are expected to constrain the matter power spectrum at percent level precision at least on scales of $k \sim 0.1 - 1 \text{ hMpc}^{-1}$ (Hearin et al. 2012).

Yet, the power spectrum, as well as higher order statistics, lose their effectiveness beyond linear scales due to the emergence of coupling among Fourier modes induced by the non-linear gravitational evolution (Rimes & Hamilton 2005; Neyrinck et al. 2006). It means that even if these future surveys can probe more deeply the non-linear regime, the amount of accessible information on the neutrino mass, or in general on other cosmological parameters, will be drastically reduced. Non-linear transformations, such as the logarithmic mapping (Neyrinck et al. 2009) or variants thereof (Seo et al. 2011; Joachimi & Taylor 2011) were introduced specifically to recapture this hidden information. Carron & Szapudi (2013), using perturbation theory, shows that this logarithmic transformation, $A = \ln(1+\delta)$, approximates well the exact “sufficient statistics” for a continuous field, meaning that it captures, by design, all the available cosmological information from the matter field. With the data analysis of future cosmological surveys in mind, investigating the constraining power of this new observable via N -body simulations with a massive neutrino component is necessary. Our goal is to extract all possible information on cosmological parameters, especially, the neutrino mass, the focus of the present work.

We proceed as follows. Section 2 presents the DEMNUni simulations. The analytical model used for the estimation of the matter power spectrum and the A -power spectrum covariance matrices is described in Section 3. Making use of this model, we forecast the Fisher information on the total neutrino mass for both the non-linear transform A and the matter power spectra. We summarize and conclude with a discussion in Section 4.

2 SIMULATIONS

The DEMNUni simulations, presented in Carbone et al. (2015); Castorina et al. (2015), are the largest N -body simulations to date with a massive neutrino component treated as an additional particle type. At present, they are characterised by a baseline Λ CDM cosmology to which neutrinos are added with a degenerate mass spectrum but different total masses. In the near future, the DEMNUni set will include also an evolving dark energy background, with different equations of state w , in order to study the degeneracy between M_ν and w , at the non linear level.

The DEMNUni simulations include only Cold Dark Matter (CDM) and neutrino particles, and have been performed using the tree particle mesh-smoothed particle hydrodynamics (TreePM-SPH) code GADGET-3 Springel (2005), specifically modified in Viel et al. (2010) to account

for the presence of massive neutrinos. These simulations have been run on the Fermi IBM BG/Q supercomputer at CINECA, Italy⁵. They contain $(2048)^3$ CDM particles and $(2048)^3$ neutrino particles, in a comoving cube of volume $V = 8 h^{-3} \text{Gpc}^3$, and are characterised by a softening length $\varepsilon = 20 \text{ Kpc}/h$. For each simulation, 62 outputs have been produced, logarithmically equispaced in the scale factor $a = 1/(1+z)$, in the redshift interval $z = 0 - 99$, 49 of which lay between $z = 0$ and $z = 10$.

The DEMNUni set is composed by four cosmological simulations sharing the same baseline cosmology consistent with the cosmological parameters estimated by the first Planck data release (Planck Collaboration et al. 2014): $\Omega_m = 0.32$, $\Omega_\Lambda = 0.68$, $H_0 = 67 \text{ km s}^{-1} \text{Mpc}^{-1}$, $\Omega_b = 0.05$, and $n_s = 0.96$. The only difference between the four simulations is represented by the total neutrino mass M_ν . One of the simulations is characterised by a pure Λ CDM cosmology without neutrinos, and is used as a reference. The other three simulations correspond to a Λ CDM cosmology plus three degenerate massive neutrinos with $M_\nu = 0.17, 0.3, 0.53 \text{ eV}$. As massive neutrinos modify the shape of the power spectrum during the cosmic evolution, a special care has been taken in setting the initial power spectrum in each simulation. The power spectrum used to set up the initial conditions has been obtained from CAMB⁶ adopting the same primordial scalar amplitude for all the four simulations. On one hand, this ensures that the normalization of the power spectrum in the four runs is consistent with Planck, on the other hand it guarantees that the shape of the initial power spectrum is consistent with the considered neutrino mass. Given the assumed Planck cosmology, the particle number, and the volume of the simulations, the CDM mass resolution is about $8 \times 10^{10} h^{-1} \text{M}_\odot$, adjusted accordingly to the value of the neutrino particle mass, in order to hold $\Omega_m = 0.32$ fixed.

3 RESULTS

3.1 Power spectra and covariance matrices

The power spectra are computed using the standard estimator defined as

$$\hat{P}(k) = \frac{1}{VN_k} \sum_{k'} |\delta(k')|^2, \quad (1)$$

where V is the survey volume and the sum runs over the N_k Fourier modes associated to the k -th power spectrum bin. We measure the real-space power spectra of both the density, P , and of the non-linear transform A , P_A , defined as

$$A \equiv \ln(1 + \delta), \quad \delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \quad (2)$$

on a 512^3 grid with the nearest-grid point density assignment deconvolving for the pixel window over the range $0.0035 \lesssim k \lesssim 0.4 \text{ hMpc}^{-1}$. The power spectrum covariance matrix is defined as

$$\text{Cov}_{ij} = \langle \hat{P}(k_i) \hat{P}(k_j) \rangle - \langle \hat{P}(k_i) \rangle \langle \hat{P}(k_j) \rangle. \quad (3)$$

¹ <http://www.darkenergysurvey.org/>

² <http://www.lsst.org/lsst/>

³ <http://sci.esa.int/euclid/>

⁴ <http://wfIRST.gsfc.nasa.gov/>

⁵ <http://www.hpc.cineca.it/>

⁶ <http://camb.info/>

Table 1. Contributions to the covariance matrix from “super-survey” and “intra-survey” modes respectively as a function of redshift. The amplitude of both parameters are estimated using the above definitions of σ_{SS}^2 and σ_{IS}^2 .

Redshift	σ_{SS}^2	σ_{IS}^2
$z = 0$	2.3×10^{-6}	1.4×10^{-6}
$z = 0.5$	1.4×10^{-6}	6.0×10^{-7}
$z = 1$	9.0×10^{-7}	3.2×10^{-7}
$z = 1.5$	5.7×10^{-7}	2.0×10^{-7}
$z = 2$	4.1×10^{-7}	1.3×10^{-7}

We follow Carron et al. (2014) who developed an approximate form of the latter in the mildly non-linear regime based on previous studies from N -body simulations (Neyrinck 2011; Mohammed & Seljak 2014)

$$\text{Cov}_{ij} = \delta_{ij} \frac{2P(k_i)^2}{N_{k_i}} + \sigma_{min}^2 P(k_i)P(k_j), \quad (4)$$

where the first term corresponds to the Gaussian covariance and the second term approximates the shell-averaged trispectrum of the field. Carron et al. (2014) have shown that the parameter σ_{min}^2 can be interpreted as the minimum variance achievable on an amplitude-like parameter. It can be further decomposed into two contributions $\sigma_{min}^2 = \sigma_{SS}^2 + \sigma_{IS}^2$, where the first term is due to the correlation between large wavelength “super-survey” modes with small scales and the second term corresponds to coupling between small scales or “intra-survey” modes.

As we consider the local density fluctuations, $\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$, defined with respect to the local observed density (i.e the global density fluctuations divided by the background mode), we have an additional contribution to the covariance matrix that can be entirely absorbed within σ_{SS}^2 . The latter can thus be expressed as $\sigma_{SS}^2 = (26/21)^2 \sigma_V^2$, where following Takada & Hu (2013)

$$\sigma_V^2 = \int P^L(k) W^2(\mathbf{k}) \frac{d^3k}{(2\pi)^3}. \quad (5)$$

In this equation, $W(\mathbf{k})$ is the Fourier transform of the survey volume window function and P^L is the linear power spectrum (Carron et al. 2014; Takada & Hu 2013, for details). Here $W(x)$ is defined as a spherical top-hat on the volume V .

The contribution to the covariance matrix coming from σ_{IS}^2 could be more complex, as it was only tested at $z = 0$ (Mohammed & Seljak 2014). However, Carron et al. (2014) derived an analytical approximation based on the hierarchical *Ansatz* (Peebles 1980; Fry 1984; Bernardeau 1996) and found that

$$\sigma_{IS}^2 \simeq \frac{P(k_{max})}{V} (4R_a + 4R_b). \quad (6)$$

We assume this expression holds for all redshifts and use it to estimate the σ_{IS}^2 contribution. Our results are presented in Table 1, and from it σ_{min}^2 is calculated using the above definition. To quantify its Fisher information content, we need an estimation of the A -covariance matrix as well. We choose to adopt a Gaussian description for the A field

$$\text{Cov}_{ij}^A = \frac{2}{N_k} P_A(k_i) P_A(k_j) \delta_{ij}. \quad (7)$$

Table 2. Information gain on neutrino mass for different redshifts. It represents the ratio between the elements of the power spectrum and A -power spectrum Fisher matrix. This gain can be interpreted as an effective gain in the survey volume.

	$z = 0$	$z = 0.5$	$z = 1$	$z = 1.5$	$z = 2$
Gain ($M_\nu = 0.235$)	8	5	4	2.5	2.1
Gain ($M_\nu = 0.415$)	6.5	4	3.5	2.5	2.1

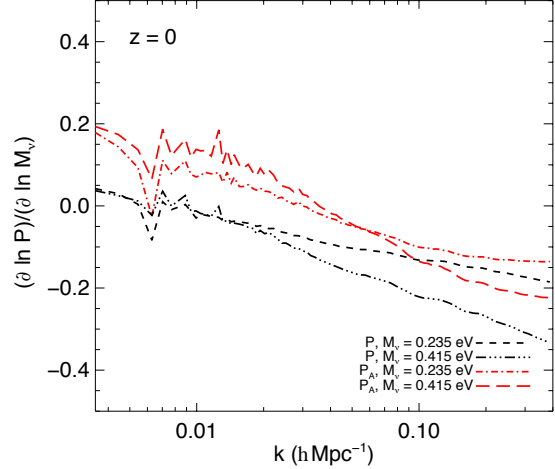


Figure 1. Logarithmic derivatives with respect to M_ν at $z = 0$ of the power spectrum (black lines) and the A -power spectrum (red lines) at two different fiducial neutrino masses.

Neyrinck (2011) have shown this form is valid to a very good approximation for $k \lesssim 0.4 \text{ h Mpc}^{-1}$, scale at which the covariance matrix of P_A starts to have non-negligible off-diagonal elements. We thus make our measurements up to this resolution limit, letting us safely assume that P_A has a diagonal covariance matrix over the full range of k considered here.

3.2 Fisher information

Given a set of parameters α, β, \dots , the Fisher matrix of the matter power spectrum is defined as

$$F_{\alpha\beta} = \sum_{k_i, k_j < k_{max}} \frac{\partial P(k_i)}{\partial \alpha} \text{Cov}_{ij}^{-1} \frac{\partial P(k_j)}{\partial \beta}. \quad (8)$$

Carron et al. (2014) have shown that the information content of the power spectrum can be written as

$$F_{\alpha\beta} = F_{\alpha\beta}^G - \sigma_{min}^2 \frac{F_{\alpha \ln A_0}^G F_{\ln A_0 \beta}^G}{1 + \sigma_{min}^2 (S/N)_G^2}, \quad (9)$$

where

$$F_{\alpha\beta}^G = \int \frac{\partial \ln P(k)}{\partial \alpha} \frac{\partial \ln P(k)}{\partial \beta} w(k) d \ln k, \quad (10)$$

with $w(k) = V k^3 / (2\pi)^2$. The discrete sums have been replaced by integrals using the fact that the number of modes N_k is approximately the surface of the shell used for the bin averaging divided by the distance element between two

discrete modes $N_k \simeq V(4\pi k^2 dk)/(2\pi)^3$. Moreover, in Equation 9, by analogy to Carron et al. (2014), we have introduced, for notational convenience, a nonlinear amplitude parameter, $\ln A_0$, defined such as $\partial_{\ln A_0} P(k) = P(k)$. This parameter corresponds to the initial amplitude σ_8^2 in the linear regime and at $z = 0$. We further define the Gaussian signal to noise as

$$(S/N)_G^2 = \int w(k) d \ln k. \quad (11)$$

Using Equation 7, the Fisher information in the A -power spectrum is given by $F_{\alpha\beta}^A = F_{\alpha\beta}^{G,A}$, where

$$F_{\alpha\beta}^{G,A} = \int \frac{\partial \ln P_A(k)}{\partial \alpha} \frac{\partial \ln P_A(k)}{\partial \beta} w(k) d \ln k. \quad (12)$$

In this analysis we focus on the information about the neutrino mass, we thus consider $\alpha = \beta = \ln(M_\nu)$ and hold fixed all the other cosmological parameters.

The derivatives in Equations 10 and 12 are calculated numerically by comparing the power spectra among simulations with different total neutrino masses. The derivatives at $z = 0$ for both the density and the A fields are shown in Figure 1 at the minimum and maximum neutrino masses considered: $M_\nu = (0.17 + 0.30)/2 = 0.235$ eV and $M_\nu = (0.30 + 0.53)/2 = 0.415$ eV.

The results are presented in Figure 2 which shows the cumulative Fisher information on the logarithm of the neutrino mass using both the power spectrum (black lines) and the A -power spectrum (red lines) at two different redshifts: $z = 0$ (left panel) and $z = 1$ (right panel). The Gaussian part of the matter power spectrum is given for comparison by the blue solid line in the case $M_\nu = 0.415$ eV. At our resolution limit, using sufficient statistics increases by up to ~ 8 the available information, leading to error bars that are almost a factor of 3 smaller compared to the matter power spectrum at $z = 0$. The evolution of the information gain with the redshift is presented in Table 2. As was shown previously by Wolk et al. (2014) in the case of projected field, the gain is larger at small redshift where the non-linearities are stronger. The fiducial value of the neutrino mass used to estimate the derivatives has almost no impact on the final gain with, however, a small tendency to have lower gain at higher neutrino mass as it can be seen on Figure 2 or in Table 2. Our results show that even at $z = 2$, the sufficient statistics unveils up to ~ 2 times more information compared to the matter power spectrum. These conclusions directly depend on the value of σ_{min}^2 and a change of 10% of the latter will result in a change of about $\sim 5\%$ in the information gain.

It can be seen on Figure 2 that in the regime $0.025 \lesssim k \lesssim 0.1 h\text{Mpc}^{-1}$, P_A is less optimal than the matter power spectrum. This is due to the bias between P_A and P that could be, in a first approximation, written as $P_A = e^{-\sigma_A^2} P_{lin}$ where σ_A^2 is the variance of the A -field (Neyrinck et al. 2009). As this bias depends on the cosmology, it shifts the 0-crossing scale of the derivatives of Equation 12 from a scale $k \sim k_{min}$ for the matter power spectrum to $k \sim 0.03 h\text{Mpc}^{-1}$ (see Figure 1), resulting in a knee in the information. Moving to larger k , the derivatives increase again in absolute value and P_A starts again to perform better than the matter power spectrum.

4 DISCUSSION

It has been extensively shown that the power spectrum is not the optimal observable to constrain cosmological parameters in the mildly non-linear regime as the information it contains saturates at a finite plateau thus contradicting the naive Gaussian expectation (Rimes & Hamilton 2005, 2006; Neyrinck et al. 2006; Neyrinck & Szapudi 2007; de Putter et al. 2012; Takada & Hu 2013). Sufficient statistics have been introduced to overcome this issue and recapture, by design, in their power spectrum most of the available information. We have demonstrated, using the DEMNUni simulations, which correspond to a Λ CDM cosmology with three degenerate massive neutrinos with sum of masses respectively 0.17, 0.3 and 0.53 eV, that the power spectrum of the non-linear transform A outperforms by a factor ~ 8 the usual power spectrum, when extracting cosmological information on the neutrino mass at $z = 0$, and by a factor of ~ 2 even at $z = 2$. This gain can be seen as an effective gain in the survey volume.

Observational issues were not considered in this work, and refinements need to be made in order to apply the non-linear transform A to actual galaxy survey data. The first one is the discreteness of galaxy surveys that must be taken into account. The effect of the shot-noise on $F_{\alpha,\beta}$ will be to change $w(k)$ into

$$w(k) = V \frac{k^3}{(2\pi)^2} \rightarrow w(k) = V \frac{k^3}{(2\pi)^2} \left(\frac{P(k)}{P(k) + \frac{1}{\bar{n}}} \right)^2, \quad (13)$$

and $P(k) \rightarrow P_g = b^2 P(k)$, where b is the linear galaxy bias. In the case of the “sufficient statistics” a new observable A^* , optimized for both the non-linearities and the observational noise, has to be used to recapture the cosmological information (Carron & Szapudi 2014; Wolk et al. 2014). Wolk et al. (2015) have derived in the case of projected fields, the information content of A^* as a function of both the information in the galaxy power spectrum and the cosmological dependencies of the bias between P and P_A . Assuming this approach holds in the 3-dimensional case, we consider the Sloan Digital Sky Survey (SDSS) LRGs sample number density, $\bar{n} \sim 5.10^{-4} h^3\text{Mpc}^{-3}$ with a galaxy bias of $b = 2$ (Percival et al. 2007; de Putter et al. 2012). At our k_{max} , we find 1σ error bars of ~ 0.01 and ~ 0.005 using P and P_A^* respectively leading to an information gain on M_ν of ~ 4 at $z = 0.3$ (corresponding to the peak of the distribution). As the clustering information content is well understood in 2D, this simple estimation is expected to give a realistic order of magnitude, however further investigations about the relationship between the matter and the non-linear transform A power spectra need to be made in 3-dimensions.

In this work, we have considered the case of the density fluctuation δ defined with respect to the local observed density as it is the case for galaxy surveys. However, in the case of fluctuations defined with respect to the global density, for example in weak lensing surveys, our predictions for the information change. The contribution from the intra-survey modes becomes negligible compared to the one from the super-survey modes, as the latter can now be written as $\sigma_{SS}^2 = (68/21)^2 \sigma_V^2$. This gives a value of $\sigma_{SS}^2 = 1.6 \times 10^{-5}$ and 3.0×10^{-6} at redshifts $z = 0$ and $z = 2$, respectively. The expected gain in information about the neutrino mass using P_A becomes about factors of ~ 25 and ~ 8 , respec-

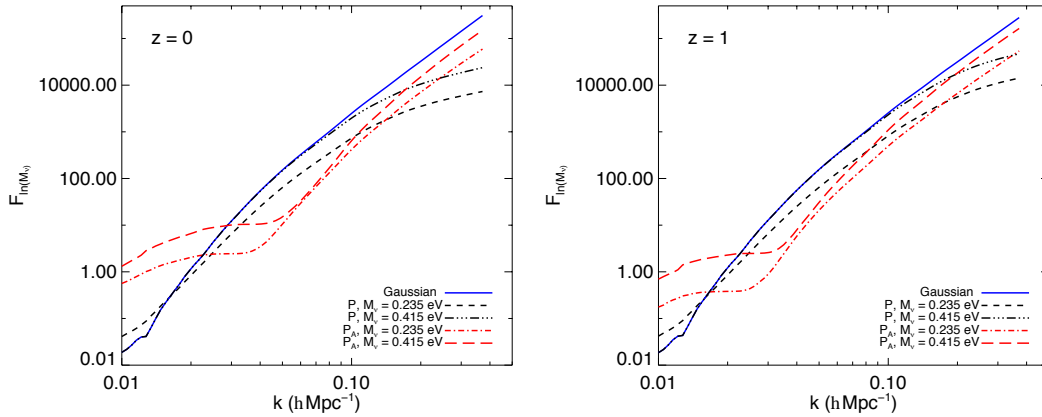


Figure 2. Cumulative Fisher information on the logarithm of the neutrino mass using both the power spectrum (black lines) and the A -power spectrum (red lines) for two different fiducial values of the neutrino mass. The covariance matrices on the two observables are given by Equation 4 and Equation 7 respectively. The Gaussian part of the matter power spectrum in the case $M_\nu = 0.415$ eV is given for comparison by the blue solid line. The left panel shows that using the “sufficient statistics” at $z = 0$ increases by ~ 8 and ~ 6.5 the available information for $M_\nu = 0.235$ eV and $M_\nu = 0.415$ eV respectively. The right panel shows the same cumulative Fisher information at $z = 1$. At this higher redshift, the gain using the “sufficient statistics” is about ~ 4 and ~ 3.5 for $M_\nu = 0.235$ eV and $M_\nu = 0.415$ eV. The evolution of the information gain with redshift is summarized in Table 2.

tively, compared to the power spectrum. This behavior was expected as the information in the matter power spectrum roughly saturates at $1/\sigma_{min}^2$. In the global case, the “super-survey” modes dominate resulting in an information plateau which is lower compared to the local case and thus allowing the non-linear transform A to perform even better.

The effect of the galaxy bias was already taken into account within the framework of the “halo model” for projected fields (Wolk et al. 2014), as well as the bias between P_{A*} and P_g . This needs to be extended to the 3-dimensional case. For future galaxy survey applications, it is also crucial to investigate the effect of redshift distortions which is left for a future work.

Despite the need of a more sophisticated analysis for a direct application to real galaxy surveys, this work has demonstrated that “sufficient statistics” are expected to be a powerful method to improve the future constraints on different cosmological parameters and, in particular, on the neutrino mass.

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